



2.2 Complex finite-dimensional vectors

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Introduction

- In this topic, we will
 - Describe complex n -dimensional vectors
 - Introduce the notation of $\mathbf{C}^n$
 - Describe zero vectors and the concept of equality
 - Discuss comparing vectors of different dimensions
 - Spoiler: just don't





n -dimensional complex vectors

- If n is a positive integer,
a complex n -dimensional vector is a sequence of n complex numbers written as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_{n-1} \\ u_n \end{pmatrix}$$

where each u_1 through u_n are complex numbers





n -dimensional complex vectors

- We will represent all n -dimensional complex vectors by

$$\mathbf{C}^n$$

- You can read $\mathbf{u} \in \mathbf{C}^n$ as “ $\mathbf{u}$ is an element of $\mathbf{C}$ to the n ”

- For example,
$$\begin{pmatrix} 1.2 + 0.3j \\ -3.7 - 1.5j \\ 6.4 + 0.0j \\ 0.8 - 8.2j \\ 9.5 + 1.5j \end{pmatrix} \in \mathbf{C}^5 \text{ and } \begin{pmatrix} 3.0 - 0.5j \\ 5.4 - 1.3j \\ 0.9 + 0.5j \\ -1.5 + 0.1j \\ 2.4 - 3.5j \\ -1.8 + 2.5j \\ 0.7 - 8.1j \\ -0.5 + 2.5j \end{pmatrix} \in \mathbf{C}^8$$





Zero vectors and equality

- The zero vector in $\mathbf{C}^n$ is still $\mathbf{0}_n$ and is the vector containing exactly n zeros: $0 + 0j$
- Two complex vectors are equal if and only if each of their n entries are equal





Real and complex vectors

- Again, we don't compare real n -dimensional vectors with complex n -dimensional vectors
 - We are either working with real vectors or complex vectors
- There are complex n -dimensional vectors, each entry of which has a zero imaginary component
 - We can say each real n -dimensional vector corresponds to a complex n -dimensional vector,
but once again: apples and oranges





Summary

- Following this topic, you now
 - Know that an n -dimensional complex vector has n complex values
 - Know the set of all n -dimensional vectors is represented as $\mathbf{C}^n$
 - Understand if $\mathbf{v}$ is a vector, then the entries are v_1 through v_n
 - Are aware that the zero vector is still the vector with all zeros
 - Understand the idea of equality is the same:
all entries must be equal
 - Understand we don't compare real and complex vectors





References

- [1] https://en.wikipedia.org/wiki/Vector_space#Complex_numbers_and_other_field_extensions





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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